VHF radar measurements and model simulations of mountain waves over Wales

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\textbf{SUMMARY}

Very high frequency (VHF) radar measurements of mountain waves over Aberystwyth, Wales, during 29 November 1997 are presented and compared with the predictions made by a three-dimensional numerical model which is based on the linearized equations of motion. The radar height–time plot of vertical velocity reveals a wave field in the troposphere and lower stratosphere which changes significantly throughout the day. Four separate radiosonde soundings taken during the day are used to represent the steady background flow in the numerical model. In this case the steady-state model wave fields are shown to compare qualitatively well with the radar measurements and the three-dimensional information provided by the model is used to help explain the changes in the wave field observed by the radar throughout the day.

The radiosonde profiles are interpolated in time to provide approximate basic-state data at two-hour intervals. Model steady-state vertical-velocity fields, based on each of these 13 approximate basic-state profiles are used to generate a model time–height vertical-velocity plot which is qualitatively similar to that obtained from the radar measurements, indicating that, to a reasonable approximation, the evolving wave field can be regarded as a sequence of independent steady states. This is generally true if the basic-state flow remains steady over the time taken for wave packets to propagate over several wavelengths. Group-velocity arguments are used to show that, for typical mountain waves in the troposphere, such time-scales are generally less than one hour.

Model simulations in which the basic-state flow is allowed to vary in time continually (based on either the interpolated radiosonde soundings or the radar horizontal-wind measurements) are conducted. In this case the simulated wave fields are always unsteady; the waves show a tendency to drift in response to changes in the basic-state wind field. For example, during instances when the wind speed at a given level decreases (whilst the wind direction remains the same) the waves show an upwind phase propagation with a typical phase speed of the order of 1 m s\textsuperscript{-1}. The consequence of this unsteadiness for the propagation of the waves is discussed.

\textbf{KEYWORDS}: Gravity waves Linear numerical model Unsteady effects

1. \textbf{INTRODUCTION}

Despite the well known importance of terrain-induced internal gravity waves (mountain waves) and their associated drag on the atmospheric circulation (e.g. Palmer \textit{et al.} 1986) there are few long-term datasets which give detailed information on their characteristics. The use of very high frequency (VHF) radar for observing mountain waves has proved to be a powerful technique for obtaining such data (e.g. Prichard \textit{et al.} 1995; Worthington and Thomas 1996; Worthington 1999); the main advantage over techniques such as radiosonde releases (e.g. Shutts \textit{et al.} 1994; Vosper and Mobbs 1996) or aircraft measurements (e.g. Brown 1983; Shutts and Broad 1993) being the ability to profile large depths of the atmosphere with a typical temporal resolution of a few minutes. However, radar measurements are usually limited to observations along a vertical, or near-vertical profile above the radar position, whose location is fixed. When trying to build up a climatology of mountain-wave behaviour this limitation can have serious consequences; one can easily imagine a case where, despite strong wave activity in the vicinity of the radar site, the waves would go undetected because of either their direction of propagation or their phase above the radar.

The recent rapid increase in computing power has allowed two- and three-dimensional (2D and 3D) numerical modelling studies of flows over realistic complex terrain. Recent numerical studies range from the finite-amplitude response to major mountain chains such as the Rockies (e.g. Clark \textit{et al.} 1994), the Pyrenees (e.g. Elkhalifi \textit{et al.}
1995; Broad 1996) and New Zealand’s Southern Alps (Lane et al. 2000) to flows over smaller-scale topography such as the UK mountain ranges (e.g. Shutts 1992; Shutts and Broad 1993; Vosper and Mobbs 1996). In most previous mountain-wave studies the wave fields are regarded as stationary relative to the ground since the background flow is assumed to be steady. Although considerable insight has been gained using this assumption, observations often show clear evidence of unsteadiness (Nance and Durran 1997; Ralph et al. 1997) and recent numerical and theoretical work has shown that this can be explained by changes in the background flow (Lott and Teitelbaum 1993a,b; Nance and Durran 1997; Ralph et al. 1997) or nonlinear wave interactions which may occur for trapped waves (Nance and Durran 1998).

The two approaches, radar measurements, which give one-dimensional profiles of the real atmosphere, and model simulations, which give a 3D but somewhat simplified picture, provide complementary, although still incomplete, insights into airflow over mountains. In this paper a specific case-study is used to illustrate how a simple 3D numerical model can be used to help interpret VHF radar measurements of mountain waves. The case presented, that of radar observations of waves over Aberystwyth, Wales, UK, on 29 November 1997, is chosen because the wave field changes considerably throughout the day and, using only radar and radiosonde measurements, it is difficult to identify the reasons for this. It is shown how the 3D fields offered by the model increase
understanding of the observations and help corroborate working hypotheses drawn from
the observations alone. The Aberystwyth VHF radar is described in section 2. A brief
outline of the numerical model is provided in section 3. The mountain-wave observa-
tions during 29 November 1997 are presented in section 4, and detailed comparisons
between the model simulations and the radar observations are made in section 5. In
section 6 some additional model simulations are performed in which the basic-state
flow is allowed to evolve in time in a realistic manner and the resulting unsteadiness in
the simulated wave field is examined. Conclusions are drawn in section 7.

2. DESCRIPTION OF THE ABERYSTWYTH MESOSPHERE–STRATOSPHERE–TROPOSPHERE
RADAR FACILITY

The Aberystwyth Mesosphere–Stratosphere–Troposphere (MST) Natural Environ-
ment Research Council (NERC) radar facility is situated near the Welsh coast at
52.42°N, 4.00°W. The location of the radar site and the surrounding topography of
central and north Wales are shown in Fig. 1. Typical mountain heights in this region
are around 500 m but rise to 1085 m in the north (Snowdon). The radar is a 46.5 MHz
VHF system with a one-way beam width of 3°, a peak-to-peak transmitted power of
160 kW and a pulse length of 8 μs, giving a height resolution of 150 m and an altitude
range of around 1.55 to 20 km. The temporal resolution is about 1.6 minutes. Four beams
are pointed 6° off the vertical in the north-west, north-east, south-east and south-west
directions and a fifth beam is pointed vertically.

3. DESCRIPTION OF THE NUMERICAL MODEL

The numerical model solves the finite-difference approximations to the linearized
3D equations of motion for an inviscid non-rotating fluid. The effect of the earth’s
rotation is neglected since it is small for the scales of motion we are interested in, the
Rossby number, \( R_o = U/fL \), being much larger than unity\(^*\), where \( U \) is a typical wind
speed, \( f \) is the Coriolis parameter and \( L \) is a horizontal length-scale. The atmosphere
is also assumed to be dry and the Boussinesq approximation is made. We should note
that the former assumption may, in certain cases, be rather restrictive since, as shown by
Durrant and Klemp (1982), the presence of deep saturated layers in the upstream flow
can significantly alter the trapped wave response. In a Cartesian coordinate system the
equations can be written as

\[
\frac{D\tilde{u}}{Dt} + \tilde{w} \frac{d\tilde{U}}{dz} + \nabla \tilde{p} - \frac{g\tilde{\theta}}{\tilde{\theta}} \mathbf{k} = \mathbf{F},
\]

\[
\frac{D\tilde{\theta}}{Dt} + \tilde{w} \frac{d\tilde{\theta}}{dz} = G
\]

and

\[
\nabla \cdot \tilde{u} = 0,
\]

where

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \tilde{U} \frac{\partial}{\partial x} + \nabla \frac{\partial}{\partial y}
\]

\( ^* \) Based on the values, \( U = 10 \text{ m s}^{-1}, f = 10^{-4} \text{ s}^{-1} \) and \( L = 10 \text{ km}, R_o = 10. \)
and
\[(u', v', w', \theta', p') = \overline{\mathbf{v}}^{-1}(\overline{u}, \overline{v}, \overline{w}, \overline{\theta}, \overline{p}).\] (5)

In the above, \(\mathbf{u} = (\overline{U} + u', \overline{V} + v', \overline{w}')\) is the velocity, where \(\overline{U}(z, t)\) and \(\overline{V}(z, t)\) are the \(x\) and \(y\) components of the basic-state velocity, respectively, and \(u', v'\) and \(w'\) are the respective \(x\), \(y\) and \(z\) wave-perturbation velocity components. The basic-state and perturbation potential temperature are denoted by \(\overline{\theta}(z, t)\) and \(\theta'\), respectively and \(g\) is the gravitational acceleration. The basic-state density and pressure perturbation are denoted by \(\overline{\rho}(z, t)\) and \(p'\), respectively. The terms \(F\) and \(G\) on the right-hand sides of Eqs. (1) and (2) represent the effects of artificial damping terms added near the model boundaries to prevent wave reflection. This damping acts on the wave perturbations only.

For the simple idealized case in which the terrain slope is small and the upstream wind speed, \(U\), and buoyancy frequency, \(N\), are independent of height and are such that the non-dimensional mountain height, \(NH/U\) is less than about unity, the linear approximation is a valid one since the wave amplitude is sufficiently small that wave breaking does not occur. The validity of the linear approximation has been explored in detail by Smith (1980, 1988). Note, however, that recent work by Nance and Durran (1998) indicates that in the case where \(U\) and \(N\) vary with height and a strongly trapped 2D wave field exists, nonlinear effects can be important even when \(NH/U < 1\).

Strictly speaking, the retention of the time dependence in the basic-state flow should cause additional terms to appear in Eqs. (1) to (3) which involve the horizontal gradients of the basic-state flow. For example, the terms \(\overline{u} \partial \overline{U}/\partial x\) and \(\overline{v} \partial \overline{U}/\partial y\) should appear in Eq. (1). However, in the case where the gravity-wave length-scale is much less than that of the background large-scale synoptic system, such terms are small and can be neglected.

Equations (1) to (3) lend themselves readily to algebraic manipulation and, in the case where the basic-state flow is steady (and terms \(F\) and \(G\) are zero), a single equation in \(\overline{w}\) can be obtained which, after Fourier decomposition of the wave field, results in a second-order differential equation for the Fourier-transformed vertical velocity, namely,
\[
\frac{d^2 \hat{w}}{dz^2} + (\ell^2 - \kappa^2) \hat{w} = 0.
\] (6)

where
\[
\ell^2(z) = \frac{N^2 \kappa^2}{(\overline{U} \cdot \kappa)^2} - \frac{1}{(\overline{U} \cdot \kappa)} \frac{d^2}{dz^2}(\overline{U} \cdot \kappa),
\] (7)

\(\hat{w}(\kappa, z)\) is the Fourier-transformed vertical velocity for a wave mode with horizontal wave vector \(\kappa = (k, l)\) and \(\kappa = |\kappa|\). In the general case where the basic-state wind and/or buoyancy frequency vary with height, this approach is complicated by the presence of trapped wave modes which arise when wave energy is partially reflected back towards the ground at levels where \(\ell^2 < \kappa^2\). Strongly trapped or resonant wave modes (whose wave energy is ducted horizontally downstream) require special treatment and a quasi-analytic solution can be found which is valid downstream of the mountain summit (Sawyer 1962; Vergeiner 1971). However, for three-dimensional flows over complex terrain this approach has a number of associated difficulties. Firstly, the resonant modes lie along lines in the \((k, l)\) plane (Sawyer 1962) which do not, in general, correspond to the discrete wave numbers resolved by a numerical model. Secondly, the resonant solution is valid downstream of the mountain summit only and no upstream information
is given. Thirdly, for arbitrary terrain there may be no single mountain summit and it is necessary to decompose the discretized terrain into individual blocks and obtain the total wave field by summing the contributions from each block (Vosper and Mobbs 1996).

Due to the above difficulties, and the fact that Eq. (6) applies to steady basic-state flows only, we shall instead adopt an alternative approach; the unsteady equations are discretized and then integrated forward in time until (in the case where the basic-state flow is steady) a steady solution is obtained. Equations (1) and (2) are discretized using a second-order accurate centred finite-difference scheme on a staggered regular Cartesian mesh. The resulting expressions are integrated forward in time from an initial state in which no waves are present using a second-order accurate modified leapfrog scheme (Rees 1988). An equation of the form

$$\nabla^2 \tilde{p} = f(x, y, z)$$  

(8)
is obtained from the divergence of the discretized form of Eq. (1) and the discretized continuity equation (3) applied at the future time step. Equation (8) closes the system and ensures Eq. (3) is satisfied throughout the integration. The technique used to solve Eq. (8) follows that used by Miranda and James (1992). A rigid-lid condition is applied at the upper boundary and Rayleigh damping (whose strength is a maximum at the upper boundary and decreases linearly to zero, typically 7–8 km below) is used to absorb upward-propagating waves and prevent wave reflection. A free-slip lower-boundary condition is applied. In linearized form this is given by

$$w' = \mathcal{U} \frac{\partial h}{\partial x} + \nabla \frac{\partial h}{\partial y},$$  

(9)

where $h(x, y)$ specifies the terrain height. It is convenient to apply this condition at $z = 0$, not $z = h$, as this allows us to solve the equations on a Cartesian grid, avoiding the need for a terrain-following coordinate system. The validity of assuming a linearized boundary condition is discussed further in section 5. Note that strict application of Eq. (9) would imply zero wave amplitude everywhere, since in reality the wind speed falls to zero at the ground. Because, however, the model equations contain no representation of surface friction, it is more appropriate to think of the model as a representation of the flow above the turbulent surface layer and apply the lower-boundary condition at some wave ‘launching height’, above the ground. Recently Worthington (1999) used VHF radar data obtained over a period of several years to demonstrate that, on average, the horizontal wave vector of mountain waves over Wales is aligned with the wind direction near the top of the boundary layer, suggesting that the waves themselves are forced at a similar height. Of course, for individual cases the appropriate choice of launching height is likely to depend on the details of the boundary-layer structure such as the thermal stability. Boundary-layer processes, not represented in this simple model, such as flow separation and convective motions may also influence the launching height. Consistent with Worthington’s (1999) results, in this study the lower-boundary condition was applied at a height of 500 m above sea level, although simulations in which the lower boundary was set to 135 m and 1000 m gave similar results.

In order to allow strongly trapped modes to propagate freely out of the domain, a radiative lateral-boundary condition, identical to the one imposed by Miranda and James (1992), is applied at the downstream $x$ boundary. Ideally such a condition would be enforced at all four lateral boundaries. However, it was found that the stability of the integrations was greatly improved when free-slip side-wall conditions were imposed
at the $y$ boundaries and a zero-perturbation condition was applied at the upstream $x$ boundary. Obviously this configuration is best suited to the case where the basic-state flow is in the $x$ direction. For situations in which this is not the case, the model domain is simply rotated around the $z$-axis so that the basic-state flow at the surface is in the positive $x$ direction. Additional thin Rayleigh-damping columns were used next to all four lateral boundaries to further minimize any wave reflections.

4. **Mountain-wave observations on 29 November 1997**

The sea-level-pressure analysis (not shown) for 12 UTC on 29 November 1997 reveals a mesoscale low centred over north-east England which gives rise to north-westerly near-surface winds over central Wales. Satellite imagery (not shown) shows low cloud cover during the day and light rain was recorded at the synoptic surface station at Bala (52.9°N, 3.6°W). The surface low was accompanied by an upper-level low which moved progressively north-east relative to the surface low during the course of the day. Consequently, the winds near the tropopause, which were light in the morning, increased significantly during the day and turned more northerly. Figure 2 shows profiles of wind speed and direction, potential temperature, buoyancy frequency and $\ell^2$ (see Eq. (7)) obtained from routine radiosonde soundings launched at 0515, 1115, 1715 and 2316 UTC from Aberporth (44 km south-west of Aberystwyth, see Fig. 1). The $\ell^2$ calculation is based on smoothed profiles of potential temperature and horizontal wind speed for a horizontal wave vector, $\kappa$, with direction 315° (i.e. pointing from north-west to south-east, thus aligned with the flow direction). The radiosonde profiles clearly show light winds at the tropopause (which occurs at around 10 km) in the early morning and increasing upper-tropospheric winds throughout the day. The potential-temperature profiles show little change with time and this is also reflected in the buoyancy frequency. There are significant changes in $\ell^2$ however; at 0515 UTC the mean value in the lower and mid-troposphere is around 0.75 km$^{-2}$ and, due to the low wind speeds, this is significantly larger near the tropopause. Throughout the day $\ell^2$ decreases near the tropopause (due to the increasing wind speed) whilst there is a slight increase in the lower troposphere. Note that we might expect the occurrence of upper-level light winds in the morning to prevent the vertical propagation of mountain waves, since the vertical component of the group velocity is reduced at levels of low wind speed (Vosper and Mobbs 1998) and wave vectors orientated such that $\mathbf{U} \cdot \kappa$ is small will exhibit critical-level behaviour (Shutts 1998). Figure 2(a) suggests that we might expect an increase in wave amplitude towards the end of the day due to the increase in the near-surface wind speed measured by the 1715 and 2316 UTC radiosondes. Note that this increase in wind speed is not reflected in surface wind measurements made about 3 km to the west of the radar site (not shown), which show a fall in the wind speed from 21 UTC onwards. However, given that the radar is located directly to the south of some major mountains (the Cader Idris range), it is possible that these surface measurements are not truly representative of the upstream flow. The trend for the wind direction in the lower troposphere to turn progressively northerly (see Fig. 2(a)) might also contribute to an increase in wave amplitude in the vicinity of the radar, since the flow becomes more perpendicular to the Cader Idris ridge to the north of Aberystwyth.

The radar measurements of the vertical and horizontal winds throughout the day are shown in Fig. 3. Consistent with the radiosonde observations, the radar observed a north-westerly flow near the surface with low wind speeds at 10 km during the morning. The wind at 10 km appeared to increase in strength and became increasingly northerly after about 10 UTC. Evidence for gravity waves can clearly be seen in the vertical-velocity
signal. Between 00 UTC and 10 UTC only a weak vertical-velocity signal was present which was almost entirely confined to the lowest 10 km. Starting at around 10 UTC, however, the amplitude began to increase and at the same time a wave signal appeared above the tropopause. Consistent with the decrease in $\ell^2$ in the lower stratosphere between 1115 UTC and 1715 UTC (see Fig. 2(c)), there appears to be a gradual increase in the vertical wavelength above 10 km which occurred between around 12 UTC and 15 UTC. This gives rise to the unusual ‘concertina’ pattern in Fig. 3(a). By 15 UTC the wave activity filled the troposphere and lower stratosphere and persisted for the rest of the day. Consistent with the previous arguments relating to the low-level radiosonde wind measurements, the strongest wave signals, with a peak-to-peak amplitude of about 1.5 m s$^{-1}$, occurred after around 18 UTC.

Although not shown here, the radar vertical-beam spectral width indicates that patches of turbulence occur intermittently between 8 and 9 km from 00 UTC to around
Figure 3. The Mesosphere–Stratosphere–Troposphere radar measurements of (a) vertical velocity (vertical beam) and (b) hourly averaged horizontal winds during 29 November 1997. Units shown in (a) are m s$^{-1}$.

16 UTC. The reasons for the existence of the turbulence are not entirely clear, but it is possibly connected with breaking of small-amplitude gravity waves beneath the critical level at 10 km. It is not clear, however, why the turbulence should persist up until 16 UTC, since we might not expect critical-level behaviour to continue much after 10 UTC, when the wind speed near the tropopause increases rapidly. High spectral-width values are also seen below 3 km throughout the day. Again, it is difficult to identify the cause of this but it could possibly be connected with the low cloud and light rain which occurred during the day.

5. Steady-state model simulations

In this section we present results from model simulations in which the background flow is considered steady. The linear model was initialized using profile data from the four Aberporth radiosonde ascents. Noise was removed from the potential-temperature and wind data by passing a three-point filter (with five passes) through the data. The
model domain consisted of $128 \times 128$ grid points in the horizontal and 60 vertical levels. The horizontal and vertical grid resolutions were 1 km and 500 m, respectively. Rayleigh damping was applied above 22.5 km in order to prevent wave reflection at the upper boundary, which was placed at 30 km. Note that since the radiosonde data generally only extended as high as 23–24 km, it was necessary to extrapolate the data with artificial values above this height to the model upper boundary. In order to promote upward wave propagation within the damping layer a constant basic-state wind (equal to that at the top of the radiosonde ascent) was imposed and the potential temperature was linearly extrapolated with a gradient which implied a buoyancy frequency equal to the average value in the uppermost 1.5 km of the profile. A separate simulation was conducted for each of the four soundings, and in each case the wave field was integrated forward in time (with a time step of 4 s) for a period of 2.67 hours, by which time a quasi-steady* wave field was obtained. Figure 4 shows the simulated vertical-velocity field at 2 km for simulations based on the 0515 and 1715 UTC soundings. Although somewhat different, a gravity-wave field is present in both cases. The structure of the overall wave pattern appears to be more coherent in the afternoon than in the morning; there is no single dominant wave vector in the latter case, whereas at 1715 UTC a coherent wavetrain, consisting of several wavelengths (of around 10 km), can be seen slightly inland of the radar site. Note also that, consistent with the increased surface wind measured by the radiosondes, the amplitude is larger in the afternoon than in the morning, the respective 0515 UTC and 1715 UTC maximum peak-to-peak amplitudes at 2 km being 4.5 and 5.8 m s$^{-1}$. The location of the strongest wave activity also appears to shift southwards during the day, from Snowdon to the Cader Idris range. This is again consistent with what we might deduce from the radiosonde wind measurements, since in the morning the low-level wind is more perpendicular to the mountain ridge in the north, whereas in the afternoon the direction has turned to be perpendicular to the mountain ridges near Cader Idris. Note that the model wave amplitudes are much larger than those recorded by the radar (see Fig. 3) and, according to the model predictions at least, this is because the radar is situated on the very edge of the wave field. This is particularly true in the morning where, despite the presence of waves with a peak-to-peak amplitude of about 2 m s$^{-1}$ over Cader Idris, relatively little wave activity is observed over the radar site itself. Figure 5 shows vertical slices through the simulated vertical-velocity fields for the 0515 UTC and 1715 UTC simulations. The exact orientation of the vertical planes is shown in Fig. 4. It appears that the morning and afternoon wave fields show an entirely different structure in the vertical. Most of the wave activity is confined to the troposphere in the morning due to the low wind speeds near the tropopause which cause critical-level absorption of the wave energy there. The increase of the upper-level winds in the afternoon, however, allows greater radiation of wave energy into the lower stratosphere and, due to the smaller values of $c^2$ in the mid and upper troposphere, there is a partially trapped response below 6 km which gives rise to large-amplitude waves, whose phase shows little variation with height.

Figure 6 shows profiles of the steady-state model predictions of the vertical velocity over the radar site for each of the four simulations. Vertical velocities deduced from the four 6° radar beams are very similar to those given by the vertical beam and so these are not shown here. The model appears to reproduce both the amplitude and the vertical wavelength of the observed waves to a reasonable degree and, consistent with the observations, the wave amplitudes are greater in the afternoon than in the morning.

* The wave fields were steady in the sense that the vertical cross-sections of the vertical-velocity field, the pressure drag and the wave momentum fluxes showed little variation in time.
Figure 4. The steady-state simulated vertical-velocity fields (m s\(^{-1}\)) at 2 km for the simulations based on the (a) 0515 UTC and (b) 1715 UTC soundings. The line gives the orientation of the vertical slice shown in Fig. 5. Also shown is the model orography (contour interval 300 m) and the location of the radar.
Figure 5. Vertical slices through the steady-state vertical-velocity fields for the simulations based on the (a) 0515 UTC (contour interval 0.4 m s\(^{-1}\)) and (b) 1715 UTC (contour interval 0.5 m s\(^{-1}\)) soundings. Solid and dashed contours denote positive and negative values, respectively. The terrain heights are shown above the \(x\)-axis and the orientation of the vertical plane is shown in Fig. 4. Note that only the lowest 25 km of the model domain is shown.

The detailed wave fields provided by the model offer support to the conclusions drawn in section 4 about the likely response of the wave field to changes in the basic-state flow.

Given that the maximum terrain height is 1085 m and in places the mountains are quite steep, the validity of the linearized free-slip lower-boundary condition is not clear. Even in the case where nonlinear phenomena such as boundary-layer flow separation do not occur, and hence the surface flow remains parallel to the terrain, the fact that the horizontal-velocity perturbations are neglected in Eq. (9) will result in a velocity field, \((U + u', V + v', w')\), at the lower boundary which is not parallel to the surface. Comparisons between the terrain slope and the slope of the flow vector at the surface (not shown), reveal, however, that significant differences only occur immediately adjacent to the steepest mountains and it seems likely, therefore, that this will have only minor consequences for the wave field in the vicinity of the radar. The role of nonlinearity in these flows, both in the equations of motion and at the lower boundary, was investigated further by using a fully nonlinear version of the current linear numerical model. This model is based on the same numerical scheme (i.e. leapfrog time integration and a centred finite-difference scheme) as the linear model, the only significant differences being that the nonlinear form of the equations are solved, the equations are transformed into the terrain-following coordinate system given by Clark (1977) and the nonlinear form of the free-slip lower-boundary condition is applied. It was also necessary to apply a fourth-order spatial filter to the velocity and potential-temperature perturbation fields every time step in order to avoid the build-up of energy at the grid scale. However, the resolved gravity-wave field was insensitive to this filter. Nonlinear simulations were conducted for the above four cases using exactly the same computational grid. Although not shown here, the nonlinear results were qualitatively similar to those given by the linear model; the amplitude in the morning simulations was small, compared with that in the 1715 and 2316 UTC simulations, and the phase and amplitude of the vertical velocity above the radar in the latter cases were similar to those given by the
linear simulations. In none of the cases did the introduction of nonlinearity significantly improve the agreement with the observations. Note also that the Fourier transform of the nonlinear and linear model vertical-velocity fields at the lower boundary were very similar for horizontal wave numbers between about 0.2 and 3 km$^{-1}$, the range important for gravity-wave forcing. This was true for all four simulations, indicating that nonlinearity in the free-slip lower-boundary condition plays only a minor role. Note, however, that the free-slip condition does not allow for representation of boundary-layer effects, in which we might expect nonlinearity to be important. Processes such as flow separation and boundary-layer convection, for example, might change the effective shape of the orography, resulting in a different wave forcing. The representation of such phenomena and their interaction with the gravity waves is beyond the scope of this study, however, and the approach adopted here of applying the free-slip condition near the top of the boundary layer should be viewed as a simple compromise.
The four Aberporth radiosonde soundings (plus the 23 UTC and 05 UTC soundings on the previous and following days, respectively) were linearly interpolated in time to provide approximate basic-state profile data at two-hour intervals throughout 29 November. The model was run to a quasi-steady state (achieved after two hours of integration time) for each of the 13 interpolated profiles. The resulting time–height plot of model vertical velocity over the radar site is presented in Fig. 7. Many of the features observed by the radar (see Fig. 3(a)) appear to be reproduced by these simulations; the waves are relatively weak and confined to the troposphere until about 10 UTC, after which the amplitude begins to slowly increase in both the troposphere and stratosphere. This sequence of simulations even appears to reproduce the way the phase lines (in time–height space) slope in the lower stratosphere after 10 UTC.

The above results indicate that in this case, to a reasonable approximation, the evolving wave field can be considered as a sequence of independent steady states. Clearly this will not always be true and the validity of this approximation must depend on the length of time it takes a steady-state wave field to develop, compared with the time-scale over which the background flow changes. The former is related to the speed at which wave packets travel (the group speed) over several wavelengths. Assuming that the basic-state flow varies slowly in the vertical, expressions for the horizontal and vertical components of the group velocity for an upwardly propagating wave packet (Gill 1982) can be used to show that the times taken to travel a horizontal and vertical wavelength, $T_x$ and $T_z$, respectively, are given by

$$T_x = \frac{2\pi N^2}{\bar{U} \kappa (N^2 \pm (N^2 - \bar{U}^2 \kappa^2))}$$  \hspace{1cm} \text{and} \hspace{1cm} T_z = \frac{2\pi N^2}{\bar{U} \kappa (N^2 - \bar{U}^2 \kappa^2)}.$$  \hspace{1cm} (10)

Note that there are two possible values for $T_x$ because the waves may propagate either into the wind or downwind. Substituting typical tropospheric values of $\bar{U} = (10, 0)$ m s$^{-1}$, $N = 10^{-2}$ s$^{-1}$ and $\kappa = 2\pi \times 10^{-4}$ m$^{-1}$ into Eq. (10) gives $T_x \sim 500$ s or 2530 s and $T_z \sim 1650$ s. Thus, if the background flow remains steady for periods of the order of an hour, we might expect that the wave field will also remain steady for that period.
Recently, Worthington (1999) has demonstrated a powerful new technique whereby VHF radar measurements can be used to infer the alignment of mountain-wave phase lines (and hence the orientation of the horizontal wave vector) above the radar. The method relies on the fact that the waves tilt the fine-scale layers of stability or humidity that occur in the atmosphere. For layers having vertical scales much smaller than the radar wavelength, the most powerful specular echoes are received from perpendicular to the layers and thus the tilting of the layers will cause imbalances of echo power between symmetric radar beams. With a sufficient number of different radar beam directions, the azimuth of the tilted layers, and hence of the mountain-wave pattern that tilts them, can be calculated. It is interesting to compare the results from this technique with the dominant wave vectors simulated by the model for this case. For each of the steady-state simulations the Fourier transform of the model vertical-velocity field was computed on every grid level. Before application of the discrete transform, a two-dimensional window function was applied to the data in order to taper the velocity field to zero at
the edges of the computational domain, thus minimizing errors associated with the non-periodicity of the model. The directions of the Fourier modes whose amplitudes were a maximum at each height are shown in Fig. 8(a). Note that the application of a 2D Fourier transform to the wave field implies that for each wave vector $|\hat{w}(-\kappa)| = |\hat{w}(\kappa)|$. In Fig. 8(a), however, only wave modes for which $k \geq 0$ are shown and thus the wave vectors tend to be roughly aligned with the wind, rather than into the wind. Figure 8(a) shows that the dominant wave-vector direction is generally between $320^\circ$ and $340^\circ$. The change towards a more westerly direction above 10 km in the early hours of the morning is presumably caused by the critical-level filtering which occurs near the tropopause. The wave-vector directions computed from the radar data are shown in Fig. 8(b). The vectors in both Figs. 8(a) and (b) were interpolated onto the same time–height grid to aid comparison. Additional vectors are displayed in Fig. 8(b) which represent the standard error in the calculation. These errors are generally large before 12 UTC due to the weak vertical-velocity signal observed by the radar, causing highly variable directions and thus we are forced to ignore the bulk of these results. More reliable measurements are obtained after 18 UTC, however, when it appears that, consistent with the simulations, the wave-vector direction is generally around north-westerly. The reasons for this are not entirely clear, though it is possible that the measurements are at times contaminated by other gravity waves, not connected with the mountains. Close inspection of the model wave fields (see Fig. 4) reveals there is a reasonable degree of spatial variability in the phase-line angle, suggesting that the radar measurements might be sensitive to its location. The calculation of the wave-vector direction from the model simulations is, however, based on the wave field across the whole model domain and so does not contain this sensitivity. Another reason for the variability in the radar-deduced wave alignments might simply be that the radar is located at the very edge of the wave pattern, where, due to the more three-dimensional nature of the wave field in this region, the scattering layers may be tilted through a wide range of azimuths.

6. Unsteady Model Simulations

We now consider the case where the basic-state profile evolves continually throughout the simulations. In order to achieve this, two different approaches were adopted. Firstly, the Aberporth radiosonde soundings were again linearly interpolated in time, but this time to give profile data at every model time step. This simulation will be referred to as the SONDE simulation. In an alternative approach (the RADAR simulation) the radar horizontal wind data were used to represent the basic-state winds. Again, this required linear interpolation onto the model time grid and at each time step the data were smoothed slightly in the vertical to remove noise. Due to the limited height range of the radar measurements (compared with the radiosondes) it was necessary to use the linearly interpolated radiosonde winds below 1.55 km and above 18 km. Since the radar provides no thermodynamic information, basic-state potential temperature and density data were also obtained from the interpolated radiosonde ascents. In both the SONDE and RADAR simulations the model was initialized with the 00 UTC 29 November basic-state data and integrated forward in time for two hours, during which time the profile was held fixed to allow an initial steady wave field to develop. This was then taken to represent the 00 UTC wave field. Integration was then continued for a further 24 hours, during which the basic-state fields were updated every time step. The resulting vertical velocities over the radar site for the SONDE and RADAR simulations are presented in Fig. 9. The time variation in both these simulations is somewhat similar to that obtained
from the sequence of steady-state model runs (see Fig. 7); both the growth of the wave amplitude and the propagation into the stratosphere during the day are captured. The two main differences between the approaches appear to be that, in the unsteady cases, stratospheric wave activity appears earlier in the day and there is an increased amount of short-time-scale variability in the troposphere (particularly after 17 UTC) than in the steady-state sequence. The first of these differences is likely to be caused by weaker critical-level absorption near the tropopause when the basic-state flow varies with time. This behaviour was also identified by Lott and Teitelbaum (1993b) and can be explained by the fact that waves with a small, but non-zero, phase speed, no longer encounter critical levels at zero-wind layers. Thus we would expect greater leakage of wave activity through the low-wind-speed layers which occur during the morning.

Visualization of the wave fields after 17 UTC shows that the high-frequency variability in the SONDE and RADAR simulations is connected with the fact that the waves have a non-zero phase speed and the phase lines drift around the domain in response to changes in the basic-state flow. From Fig. 9 it is clear that the RADAR simulation...
Figure 10. Horizontal cross-sections of vertical velocity (m s$^{-1}$) at 2 km during the unsteady SONDE simulation at (a) 1745 UTC and (b) 1933 UTC. Also shown are terrain-height contours (interval 300 m) and the position of the radar. Only the region close to Aberystwyth is shown.
exhibits variations on a shorter time-scale than does the SONDE simulation. This is presumably due to the more rapid variation in the radar wind measurements, although these rapid changes may themselves be connected with unsteady gravity-wave motion over Aberystwyth rather than a genuine feature of the upstream flow. An example of the wave unsteadiness is presented in Fig. 10, which shows the vertical-velocity fields at a height of 2 km in the SONDE simulation at 1745 UTC and 1933 UTC. Between these times the phase lines exhibit a pronounced upstream propagation. At 1745 UTC the point 2 km above the radar site (marked in Fig. 10) lies in a region of positive vertical velocity and, due to the upstream movement of the phase lines, the phase of the waves has changed through approximately 180° by 1933 UTC, consistent with an upstream phase speed of about 0.8 m s\(^{-1}\). Examination of Fig. 2 reveals that this upstream phase propagation coincides with a decrease in the wind speed over the lowest few kilometres. At 2 km, the interpolated basic-state wind speed used in this simulation actually falls by about 1.1 m s\(^{-1}\) over this period, whilst the wind direction changes by less than 4°. This behaviour is consistent with results from an idealized two-dimensional study by Nance and Durran (1997), who showed that changes in the basic-state wind speed can give rise to a non-stationary wave field. A decrease (increase) in the basic-state wind will cause previously stationary waves (whose phase speed was equal and opposite to the basic-state flow) to drift upwind (downwind) with a phase speed equal to the change in the basic-state wind speed. In three dimensions this effect is further complicated by changes in the basic-state wind direction over time. Although not shown here, both the SONDE and RADAR simulations show clear evidence for a slight sideways drift towards the south-west (along the direction of the phase lines) of the entire wave field at 2 km between 10 and 15 UTC, as the basic-state flow direction becomes progressively more northerly.

7. Conclusions

Linear numerical-model simulations of mountain waves over Wales during 29 November 1997 were compared with high-resolution VHF radar observations. The model steady-state vertical-velocity profiles above the radar agree qualitatively with the observations, and the simulations show clearly that the character of the waves changes significantly during the day. In the morning the simulated wave fields exhibit critical-level behaviour near the tropopause and thus there is very little propagation into the stratosphere and critical-level absorption prevents downward reflection of wave energy back into the troposphere. During the afternoon, however, an increase in the wind speed near the tropopause allows both upward radiation of wave energy into the stratosphere and partial reflection of wave energy back down into the troposphere. At the same time an increase in the northerly component of the near-surface wind contributes to an increase in the wave amplitude near the radar. The detailed wave fields provided by the model simulations support the hypotheses drawn from the data and the spatial structure of the model wave fields allows greater insight than is available from the radar measurements alone. For example, without the model wave fields it would not be entirely clear why so little tropospheric wave activity is observed by the radar during the morning. Part of the reason is simply that, at that time, the radar lies on the very edge of the wave field and the propagation of the wave energy is such that it is absorbed at 10 km rather than being partially reflected back down into the troposphere.

By interpolating radiosonde measurements in time to give approximate basic-state data throughout the day, it has been shown how linear steady-state theory can provide a qualitative description of the evolving wave field as a sequence of independent steady
states. Group-velocity arguments indicate that this will only be true if the background state remains steady on time-scales of the order of one hour. Model simulations in which the basic-state profile is allowed to vary in time are qualitatively similar to the sequence of steady states. Unsteadiness in the basic-state flow gives rise to two main differences however: critical-level absorption is weaker and thus there is greater vertical propagation of wave energy than in the steady-state case and, relative to the ground, the waves are generally non-stationary. The latter is particularly true in the troposphere where, during the afternoon, the phase lines of the partially trapped waves can be seen to move around in response to changes in the basic-state wind. Phase speeds are typically around 1 m s$^{-1}$ and the direction of propagation may be upwind, when the basic-state wind speed decreases with time, downwind, corresponding to an increase in the wind speed or, when the basic-state wind rotates, the whole wave pattern may drift along in the direction of the phase-line orientation. Note that we make no claim here that these unsteady simulations provide a better representation of the observed waves than do the steady-state simulations. Indeed, calculation of a normalized error function based on the mean absolute difference between the model and radar velocities, indicates that the inclusion of the time-dependent basic-state flow does nothing to improve the overall accuracy. The exact reason for this is difficult to identify and perhaps, given the simplicity of the model, we should not be surprised at this discrepancy. However, one possibility is that, in the SONDE simulation at least, the linear interpolation (in time) of the radiosonde data implies that the basic-state wind field is never steady. Thus, the waves themselves can never be entirely stationary, which could lead to an over prediction of the unsteadiness of the wave field. If this were the case, we might expect the agreement to improve when a more accurate representation of the evolution of the basic-state wind is used. Using the radar winds to drive the model (the RADAR simulation) does not improve matters however and thus, for the time being, the precise reason for this discrepancy remains unclear.

Despite the above uncertainties the unsteady simulations do highlight an interesting phenomenon which could have important implications for gravity-wave drag parametrization: both the vertical propagation of mountain waves into the stratosphere and the horizontal propagation of trapped waves through the troposphere might be affected significantly by a rapidly varying mean flow. Note that Nance and Durran (1998) have demonstrated that for 2D trapped waves, even when the basic-state flow remains steady, the waves may still exhibit significant unsteadiness due to nonlinear wave interactions. This is true even for small-amplitude waves. The importance of this in 3D wave fields is currently not clear but, given especially that trapped waves often exhibit a quasi-2D structure, it seems likely such effects may contribute to further wave unsteadiness in real flows.

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REFERENCES


