Measurement of the pressure field on a mountain

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(Received 8 August 1995; revised 28 May 1996)

SUMMARY

Four microbarographs, accompanied by wind vanes and anemometers, were deployed on a mountain called Black Combe (height 600 m) in Cumbria during a field experiment that took place in November 1991. The aims of the experiment were to measure the small flow-induced pressure differences across the mountain and relate these to the local wind. These pressure differences may be used to calculate directly the drag exerted by the atmosphere on Black Combe.

The mean pressure difference between pairs of sites calculated over periods when the wind speed is small is due to the hydrostatic pressure difference caused by the height differences between the instruments. Removing this component reveals pressure differences across the mountain of up to 2 hPa. The main result of the experiment is the high correlation of the pressure differences with differences of the quantity \( p \frac{u^2}{2} \), where \( p \) is the air density and \( u \) is the wind speed. It is shown that this result holds for a stratified fluid when far upstream the streamlines originate from levels of similar wind speed.

Although reliable data were only obtained at three of the four stations, making the assumption that the dynamic pressure varies linearly across the mountain surface enables an estimate of the drag to be made. The average drag exerted on Black Combe over the period of the experiment is estimated to be 3.6 Pa.

KEYWORDS: Pressure drag Microbarograph data Orography

1. INTRODUCTION

In recent years it has been recognized that internal gravity waves play an important role in the atmospheric global momentum budget. The incorporation of gravity-wave-drag parametrization schemes into numerical weather-prediction and climate models has proved generally beneficial in that systematic biases towards westerly winds in the northern hemisphere are much reduced (e.g. Palmer et al. 1986; McFarlane 1987). These schemes generally assume that the waves generated by subgrid-scale orography are hydrostatic in nature and are able to propagate freely in the vertical. The effects of trapping on shorter wavelengths are generally ignored in such schemes. Although the latest parametrization schemes now attempt to incorporate other gravity-wave processes there is nevertheless a need for further study at the smaller scales.

Much attention has been directed towards numerical modelling of internal gravity-wave generation over realistic terrain (e.g. Vergeiner 1971; Klemp and Lilly 1978; Peltier and Clark 1979; Clark and Gall 1982; Shutts and Broad 1993). Such an approach can provide both very detailed spatial and temporal information regarding the wave structures and associated drag. However, there is currently a need for further detailed observations so that the numerical results can be validated.

There are generally two ways of determining the magnitude of the drag exerted on the atmosphere by the generation of gravity waves. The first method involves making observations of the atmosphere and determining the vertical fluxes of horizontal momentum. This can be achieved in a variety of ways such as radiosonde observations (e.g. Corby 1957; Reid 1972; Shutts et al. 1994), aircraft measurements (e.g. Lilly and Kennedy 1973; Lilly 1978; Brown 1983; Shutts and Broad 1993) and ground-based radar profiling (although the latter is rather restricted by the location of the instrumentation). The second method is to measure the small but significant pressure differences across the orography and apply Newton’s third law, i.e. the force exerted on the atmosphere by the orography must be equal and opposite to the force exerted on the orography by the atmosphere. If the spatial

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resolution of the pressure measurements is high enough then a direct drag calculation can be made.

In recent years there have been a number of attempts to record the pressure differences and drag across large-scale mountain ranges. Notably, Hafner and Smith (1985) computed the drag as a two-dimensional horizontal vector for the European Alps, using surface pressure data provided by synoptic stations during ALPEX. They divided the Alps into four horizontally separated regions and then subdivided each region into three distinct vertical layers. The drag was estimated for each of these regions. Davies and Phillips (1985) examined surface pressure data in one of these regions (the St. Gotthard section) and estimated the mean drag exerted by the atmosphere on this region to be 0.78 Pa. Peak values of the drag were estimated to be ±7.5 Pa. Carissimo et al. (1988) estimated the drag exerted on the entire Alps using surface pressure measurements obtained during ALPEX also. They used different methods to previous authors to obtain their estimates and found peak values of the drag to be slightly smaller than those estimated by Davies and Phillips (1985), but larger than those suggested by Hafner and Smith (1985). ALPEX surface pressure measurements were used to calculate the drag on the Dinaric Alps (situated on the northern Adriatic coast) by Tutis and Ivancan-Picek (1991). They concluded that 'there is a major sink of atmospheric momentum over the Dinaric Alpine region'. The drag exerted by the atmosphere on the Pyrénées was computed at half-hourly intervals during PYREX by Bougeault et al. (1993). Pressure measurements were made at 15 stations on a transect perpendicular to the mountain range, and the measured drag ranged from about −7 to +8 Pa. Smith (1978) conducted an experiment on the Blue Ridge mountain in the central Appalachians. This mountain is on a much smaller scale (height 300 m, width 2.5 km) than the mountains mentioned above and it is essentially a two-dimensional ridge with a length of about 35 km. For such a mountain the flow can be regarded as approximately two-dimensional except near the ends of the ridge. In Smith's study, seven microbarographs were deployed on the Blue Ridge in an attempt to measure pressure differences across the mountain. He recorded pressure differences of between 0.2 and 0.5 hPa and deduced that the two most important mechanisms causing the associated drag (which was estimated to be around 4 Pa) were the upstream blocking of cold low-level air and the generation of internal gravity waves. The near-surface flow over small hills of moderate slope has also been investigated in a number of studies (e.g. Mason and Sykes 1979; Mason and King 1985). For such small hills (e.g. the hill studied by Mason and Sykes had a height of 137 m and a width of about 1200 m) the dominant drag mechanisms are likely to be turbulent dissipation (Belcher et al. 1993) and flow separation, rather than the generation of gravity waves.

This paper describes a field experiment which aimed to measure the pressure differences across a mountain. The mountain chosen for this study is similar in width to the Blue Ridge mountain considered by Smith (1978); it and has a height of about 600 m and a half-width of around 2.5 km. The main difference between Smith's study and the one presented here is that the mountain is truly three-dimensional, and thus the air can more easily flow around as well as over the mountain.

2. AIMS AND LOCATION OF THE EXPERIMENT

The aims of the experiment were primarily to study the flow over and around an isolated mountain and to make direct measurements of the small but significant pressure differences between the upwind and downwind sides. These pressure differences can then be related to the local wind and also used to estimate the drag force on the mountain.
The mountain chosen for this study was Black Combe, located on the Cumbrian coast in north-west England (54°16'N, 3°19.7'W). It has a height of 600 m and is reasonably isolated from other orography when the incident flow has a westerly component. A contour map showing the relevant region of north-west England is presented in Fig. 1. The unobstructed exposure to westerly winds, and the three-dimensionality of the mountain are the main reasons for choosing it for this study. The mountain surface is covered mainly in short grass, bracken and heather and is rather flat near its summit. It has an average slope ($\delta z/\delta x$) of about 1/4.

3. Experimental methods

Between 20 and 29 November 1991, four microbarographs were deployed at different sites on Black Combe. Each microbarograph was accompanied by a wind vane and an anemometer on a 2 m mast. The locations of the four stations (referred to as stations 1, 2, 3 and 4 henceforth) and a more detailed view of Black Combe are shown in Fig. 2. Station 3 is actually situated on an adjoining mountain called White Combe, approximately to the east of Black Combe. These sites were chosen because it was considered that they would be representative of upwind and downwind locations for a wide range of incident wind directions. They were also convenient sites to visit for data collection. Approximate heights above sea level for each of the four stations are given in Table 1.

<table>
<thead>
<tr>
<th>Station 1</th>
<th>Station 2</th>
<th>Station 3</th>
<th>Station 4</th>
</tr>
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<tbody>
<tr>
<td>480 m</td>
<td>365 m</td>
<td>290 m</td>
<td>350 m</td>
</tr>
</tbody>
</table>

Note that the given heights are only approximate.
Figure 2. Contours showing the orographic heights (in metres) for Black Combe and White Combe. The positive $y$ direction is north. The approximate positions of stations 1, 2, 3 and 4 are denoted by crosses. Station 3 actually lies on White Combe. The positions of the stations were determined by triangulation.

It must be stressed here that the heights shown in Table 1 are only approximate and without using expensive satellite or surveying techniques it is almost impossible to know the exact position and height of a location. These heights are therefore only guidelines based on bearings taken with a compass and the careful study of a map in the field. The fact that we do not know the exact heights (and hence we cannot place instruments at equal heights on the upwind and downwind sides of a mountain) means that there will always be a large pressure difference between any two stations due to the height differences in their locations. This large hydrostatic component is at least an order of magnitude greater than any pressure difference due to a dynamical effect and must therefore be removed. The technique used was to calculate a mean pressure difference between each pair of stations over periods when the average wind speed was below a certain critical value. At these times we would expect the pressure difference to be due to the height differences only, and that the dynamical contribution should be small. Ideally, we would like to measure the pressure difference when there is no oncoming flow at all. However, since these conditions were not encountered during the experiment it has been necessary to use a value of 6 m s$^{-1}$ as the critical wind speed (note that 6 m s$^{-1}$ corresponds to a dynamic pressure, $\frac{1}{2} \rho u^2$ where $u$ is the wind speed and $\rho$ is the air density, of about 0.2 hPa). When the critical wind speed was increased towards 10 m s$^{-1}$, the effects of dynamical processes started to become apparent. When it was decreased to as little as 2 m s$^{-1}$, the averages became unreliable because of the small amount of data remaining. Between 4 and 8 m s$^{-1}$ the averages were relatively insensitive to the critical wind speed.

4. **TECHNICAL DETAILS OF THE EXPERIMENTAL SET-UP**

The microbarographs used to measure the surface pressure are exactly those described by Monserrat et al. (1991). They are described briefly in this section.
The microbarographs use a differential pressure transducer with a pressure range of $\pm 20$ hPa and an output voltage sensitivity of about 100 mV hPa$^{-1}$. Monserrat et al. (1991) measured the noise in the pressure transducer output as less than 1 mV root mean square (r.m.s.). This implies an r.m.s. error in pressure of less than 0.01 hPa.

One input to the pressure transducer was connected to a reference-pressure oven which consisted of a vacuum flask that was maintained at an almost constant temperature ($\approx 35 ^\circ$C) by using a mercury contact thermometer. The heating of the oven was powered by an 80 Ah heavy-duty lorry battery. The pressure transducer was itself kept at a constant temperature (in order to avoid any effects of temperature on the output voltages) by housing it in the thermostatted oven, which was in turn stored in an insulating ‘cool box’ along with the rest of the electronic equipment.

The other pressure transducer port was connected via 3 to 4 m of strong nylon piping to a static pressure head. This was designed in such a way that atmospheric pressure could be measured without the very local effects (i.e. immediately around the pressure head) of fluctuations in the wind speed. The pressure head consisted of a flat circular metal plate (of 150 mm radius) with a circular hole in the centre (of diameter 5 mm). The nylon piping was connected to a short vertical pipe beneath the hole in the plate which was open at the bottom. This was then pushed flush into the ground, the idea being that any rainwater would run through the central hole and drain into the ground, not affecting the pressure readings. A schematic diagram illustrating the experimental set-up is presented in Fig. 3.

The pressure measurements were recorded as 30 s averages using a data-logger. Wind speed and direction measurements were also recorded at each station. These data were recorded every 5 minutes. At this logging rate, 46 hours of data could be stored. Every day the data were downloaded from the data-loggers at each station onto a portable computer.

The reliability of the microbarographs was investigated after the experiment by running them side by side in a laboratory for a period of about two weeks. The agreement between the pressure data recorded by each instrument was very good; the differences in pressure readings being less than 0.1 hPa.
5. Technical difficulties

There are various reasons why we might not expect the measured pressure differences to be due to the dynamical effect of wind blowing over and around the mountain alone. One possibility is that due to the geostrophic wind balance, a strong wind will have an associated pressure gradient perpendicular to the wind direction. This pressure difference will contribute to the pressure differences measured on the mountain. However, a wind of 25 m s\(^{-1}\) will cause a geostrophic pressure difference at a latitude of 52° of about 0.1 hPa for microbarographs placed 4 km apart. It seems that this effect is therefore quite unimportant.

The main source of error is due to the possible temperature differences on different slopes of the mountain. When there is little or no air motion, differential heating of slopes by the sun may cause significant temperature, and hence pressure, differences across the mountain. This is more likely to happen in the early morning or late afternoon, when there is a low solar elevation. Smith (1978) considered a two-dimensional ridge for which the air temperature below the ridge summit was different on the two sides of the ridge. By the hydrostatic relation, the pressure difference across the ridge, \(\Delta p\), is given by

\[
\Delta p \approx \frac{gH\bar{\rho}\Delta T}{2T},
\]

where \(H\) is the ridge height, \(\bar{\rho}\) is the air density averaged across the width of the ridge, \(T\) is the average temperature, \(\Delta T\) is the temperature difference across the ridge and \(g\) is the gravitational acceleration. Values of \(H = 600\) m, \(\bar{\rho} = 1.3\) kg m\(^{-3}\), \(T = 300\) K and \(\Delta T = 2\) degC would imply a pressure difference of about 0.25 hPa, and thus it seems that this effect could be quite important.

6. Results of the experiment

Figures 4(a), (b) and (c) show the wind vectors measured at stations 1, 2 and 4 respectively (only one vector every 50 minutes is displayed). Note that the wind directions generally differ from each other by less than 10° for most of the time. Unfortunately, due to a problem with the instrumentation at station 3, no reliable data were obtained there. Extensive testing after the experiment has revealed that the problems at this site were probably due to heavy rainfall entering the nylon pipe to the microbarograph via the static pressure head. The design of the pressure head has consequently been improved to avoid this problem in any future experiments. Note, also, that the gap in the data collected at station 1 is due to a problem with moisture that accumulated inside the data-logger. Despite these difficulties, examination of the reliable data reveals that the wind speed at station 4 is up to twice that observed at stations 1 and 2 for most of the experiment. A simple explanation for this might be that station 4 is positioned on the leeward side of the mountain for most of the duration of the experiment. If upwardly propagating internal gravity waves are being generated by the mountain then the associated low pressure on the leeward side of the mountain, and high pressure on the windward slope (and hence the net drag), will accelerate the air on the leeward side of the mountain. Another possibility is that despite the careful siting of the instrument, a very local speed-up effect is taking place at station 4, due to some local feature of the mountain surface.

During the course of the experiment, radiosondes were released from the UK Meteorological Office station at Eskmeals (see Fig. 1). Sondes were launched frequently from Eskmeals as part of another field experiment (Vosper and Mobbs 1996), and for flow with a westerly component the profiles obtained are representative of the conditions upstream.
MEASUREMENT OF THE PRESSURE FIELD ON A MOUNTAIN

Figure 4. The wind vectors measured at (a) station 1, (b) station 2 and (c) station 4 (see Table 1) during the experiment. The time shown on the horizontal axis refers to the time in hours after 0000 GMT on 20 November 1991. Gaps in the data correspond to where there were no measurements (due to technical problems) or where corrupted data have been removed.

of Black Combe. The Froude number, $F = U/NH$, where $U$ is the upstream wind speed and $N$ is the buoyancy frequency, has been calculated by averaging the values of $U/N$ measured below the summit of the mountain, for each of the available profiles during the period of the experiment*. The variation of $F$ with time is shown in Fig. 5. Typical values of $F$ appear to be around 2 with values as high as 4.9 and as low as 1.2. Examples of the upstream wind and stability profiles encountered during the experiment are presented in Fig. 6 which shows (a) the wind speed variation with height and (b) the variation of $N$ in the lowest 3 km. The solid lines depict the wind speed for the lowest Froude number case encountered ($F = 1.2$, 62.3 hours into the experiment) and the dashed line shows the profile for the highest value of $F$ ($F = 4.9$ at 153.9 hours).

Averaging the pressures at each of stations 1, 2 and 4 using measurements when the wind speed was simultaneously less than 6 m s$^{-1}$ at all three stations gave the average surface pressures 966.1 hPa, 980.8 hPa and 985.5 hPa for stations 1, 2 and 4 respectively. Figure 7 shows the variation of the pressure recorded at station 1 minus the pressure at

* This averaging strategy was adopted since it is otherwise unclear what the relevant values of $U$ and $N$ are in calculating the Froude number due to the variation of wind speed and stability with height in the atmosphere.
Figure 5. The variation of the Froude number, $F$, during the experiment as calculated from radiosondes released from Eskmeals. The time shown on the horizontal axis refers to the time in hours after GMT on 20 November 1991.

Figure 6. The variation of (a) wind speed and (b) buoyancy frequency with height as obtained by radiosondes for the minimum Froude number case (solid lines) and the maximum Froude number case (dashed lines).

station 2 (solid line) after the component of the pressure differences due to the different heights of the instruments has been removed. Unfortunately, due to the problem with the data-logger at station 1 (caused by moisture) very little data were recorded at this site after 85 hours. Nevertheless, we can see a dynamical pressure difference of just under 0.5 hPa. Also shown is the quantity $\frac{1}{2} \rho (u_2^2 - u_1^2)$ where $u_i$ denotes the wind speed at station $i$ and $\rho$ is assumed to be 1.3 kg m$^{-3}$ (this assumption was necessary as no temperature measurements were made at the sites). There appears to be a close correlation between the pressure and $\frac{1}{2} \rho u^2$ differences during the first 85 hours of the experiment. The degree of this correlation can be more objectively tested by calculating a normalized cross-correlation coefficient, $\Sigma^2$, where

$$\Sigma^2 = \frac{\Delta p' \{ \Delta (-\rho u^2/2) \}}{\sqrt{(\Delta p')^2 \{ \Delta (-\rho u^2/2) \}^2}}, \tag{2}$$

where $\Delta p'$ is the pressure difference (after removal of the component due to the height differences between the instruments), $\Delta (-\rho u^2/2)$ is the difference in $\frac{1}{2} \rho u^2$, and the overbar denotes integration over time. Values of $\Sigma^2$ close to 1 indicate a positive correlation,
values close to 0 indicate no correlation, and values near -1 indicate the two signals are 180° out of phase. Calculating $\Sigma^2$ over all the reliable data obtained from stations 1 and 2 gives a value of 0.67, showing that the pressure and $\frac{1}{2}\rho u^2$ differences are indeed well correlated.

In a similar way to Fig. 7, Figs. 8 and 9 show $\Delta p'$ and $\Delta (-\rho u^2/2)$ between stations 1 and 4, and stations 2 and 4, respectively. In both these figures gaps in the data signify that corrupted data have been omitted. This corruption was mainly due to severe weather conditions (e.g. heavy rain entering the tube connecting the static pressure head to the pressure sensor). However, the remaining reliable data show that there appears to
be a pressure difference between pairs of stations with magnitude up to about 2 hPa. Furthermore, this pressure difference is closely correlated with the differences in $\frac{1}{2} \rho u^2$ with values of $\Sigma^2$ of 0.88 for stations 1 and 4 and 0.94 for stations 2 and 4.

Smith (1980) has calculated the surface-pressure perturbation for orographically forced linear internal gravity waves over a three-dimensional bell-shaped mountain whose profile is given by $h(x, y) = H/(r^2/a^2 + 1)^{3/2}$, where $a$ is the mountain half-width and $r$ is the radial distance from the summit. The surface-pressure perturbation for flow over such a mountain is

$$p'(x, y) = -\rho U N H \frac{x/a}{(1 + r^2/a^2)^{3/2}}.$$  \hspace{1cm} (3)

Strictly speaking, the derivation of Eq. (3) requires the assumption that the flow is in hydrostatic balance (i.e. that $U/Na \ll 1$). In general this will only be true for mountains which are much wider than Black Combe. Unfortunately, the surface pressure for flows which are not in hydrostatic balance cannot be computed analytically. Nevertheless, inserting the values $\rho = 1.3 \text{ kg m}^{-3}, U = 15 \text{ m s}^{-1}, H = 600 \text{ m}$ and $a = 2.5 \text{ km}$ gives a maximum value of 0.45 hPa for the surface-pressure perturbation. This implies a maximum pressure difference across the mountain of 0.9 hPa. This is of the same magnitude as the pressure differences measured during the experiment.

7. **Analysis of the Pressure and $\frac{1}{2} \rho u^2$ Result**

By Bernoulli’s theorem, for the steady flow of an inviscid fluid in isentropic motion, the quantity $I$, where

$$I = \frac{u^2}{2} + h + gz,$$ \hspace{1cm} (4)

is constant along a streamline. Here, $u$ is the velocity vector, $h = c_p T$ is the specific enthalpy, $c_p$ is the specific heat capacity of air at constant pressure and $z$ is the height. We now consider two streamlines with associated ‘Bernoulli constants’ $I_1$ and $I_2$ that pass through points labelled 1 and 2 on the mountain. This is illustrated in Fig. 10.
Far upstream of the mountain we assume that the flow is horizontally homogeneous and that the streamlines are a small distance $\delta_u z$ apart in the vertical. Then

$$\delta_u I = \delta_u \left( \frac{1}{2} u^2 \right) + \delta_u h + g \delta_u z$$

since enthalpy can be related to the entropy (denoted by $s$) by $\delta h = T \delta s + \delta p / \rho$. In Eq. (6) the subscript $u$ denotes that the quantities are evaluated upstream. Far upstream we assume that the flow is unaffected by the mountain and is in hydrostatic balance, hence,

$$\frac{1}{\rho} \delta_u p \approx -g \delta_u z.$$  

Substituting expression (7) into (6) then gives

$$\delta_u I = \delta_u \left( \frac{1}{2} u^2 \right) + T \delta_u s.$$  

Now, on the mountain surface,

$$I_1 - I_2 = \frac{1}{2} (u_1^2 - u_2^2) + T (s_1 - s_2) + \frac{1}{\rho} (p_1 - p_2) + g (z_1 - z_2),$$

where the subscripts 1 and 2 denote quantities evaluated at points 1 and 2 on the mountain. Since entropy is conserved on the streamlines (assuming that boundary-layer dissipation and other dissipative processes are negligible),

$$T (s_1 - s_2) \approx T \delta_u s.$$  

We now write $p_1 = \overline{p}_1 + p'_1$ and $p_2 = \overline{p}_2 + p'_2$, where $\overline{p}_1$ is the mean pressure at site 1 (due to the height of point 1 above sea level) and $p'_1$ is the perturbation to this pressure due to the effect of the mountain on the flow. This applies similarly for point 2. Since the mean pressures $\overline{p}_1$ and $\overline{p}_2$ are hydrostatic, we can write $\overline{p}_1 - \overline{p}_2 \approx g \rho (z_2 - z_1)$; substituting this along with expression (10) into Eq. (9) then gives

$$I_1 - I_2 = \frac{1}{2} (u_1^2 - u_2^2) + T \delta_u s + \frac{1}{\rho} (p'_1 - p'_2).$$
Equating expressions (11) and (8) then gives the required result, namely,
\[ \frac{1}{2} (u_1^2 - u_2^2) + \frac{1}{\rho} (p_1' - p_2') + \delta u \left( \frac{1}{2} u^2 \right) = 0 . \] (12)

Thus, it seems that if the third term on the left-hand side of Eq. (12) is small compared with the other terms then even though the fluid is stratified the difference in \( \frac{1}{2} \rho u^2 \) will be directly related to the pressure difference. In other words, if far upstream the streamlines originate from levels of similar wind speed, then even though the two points on the mountain may not lie on the same streamlines, the pressure and \( \frac{1}{2} \rho u^2 \) differences will be related. An important consequence of this result is the suggestion that it may not be necessary to actually measure the pressure on the mountain, and that measuring the differences in wind speed may be enough to determine the pressure differences.

The fact that for many of the stations the \( \frac{1}{2} \rho u^2 \) and pressure differences, although highly correlated, differ by an approximately constant value may be an indication that we have not correctly removed the mean pressure due to the height differences of the instruments. Unfortunately, it is not practical to eliminate the hydrostatic pressure differences using a critical wind speed much less than 6 m s\(^{-1}\) with the data we obtained since the number of records for when this occurred is very small (e.g. there is only 10 minutes of data for which the wind speed at all four stations is less than 4 m s\(^{-1}\)).

8. AN ESTIMATE OF THE DRAG

In this section an estimate of the horizontal drag exerted on Black Combe is made using the pressure measurements collected during the course of the experiment. We need to make some simplifying assumptions about the pressure distribution in order to do this because the spatial resolution of the pressure measurements is low compared with our knowledge of the orography.

Along the lines of Hafner and Smith (1985), the drag vector, \( \mathbf{F} \), is given by
\[ \mathbf{F} = -\int_S p \mathbf{n} \, dA = -\int_V \nabla p \, dV , \] (13)

by the divergence theorem, where \( \mathbf{n} \) is the normal vector to the mountain surface \( S \), \( A \) is the surface area of the mountain and \( V \) is its volume. We are only interested in the horizontal component of this vector, and assuming that the horizontal pressure gradient is constant we can take it outside the integral and write
\[ \mathbf{F}_H = -\nabla_H pV , \] (14)

where the subscript \( H \) indicates the horizontal component. Then,
\[ p_1' = p_2' + (x_1 - x_2) \frac{\partial p'}{\partial x} + (y_1 - y_2) \frac{\partial p'}{\partial y} \] (15)

and
\[ p_1' = p_4' + (x_1 - x_4) \frac{\partial p'}{\partial x} + (y_1 - y_4) \frac{\partial p'}{\partial y} , \] (16)

where \( p_i' \) and \((x_i, y_i)\) are, respectively, the pressures after removing the mean pressure at low wind speed and Cartesian coordinates of station \( i \). From Eqs. (15) and (16) we can
now determine $\partial p'/\partial x$ and $\partial p'/\partial y$ and hence, using Eq. (14), the horizontal component of the drag. The volume of Black Combe was estimated using the data set that is contoured in Fig. 2. This is approximately $8.8 \times 10^9$ m$^3$. The drag was calculated using all the data that were available from stations 1, 2 and 4 simultaneously during the first 85 hours of the experiment. Figure 11 shows the drag vectors during this period.

The average drag magnitude per unit area is 6.4 Pa. The directions of these drag vectors appear to be of some concern. From Fig. 11 we can see that most of the vectors with significant amplitude are pointing from the south-east. This is quite different to the wind directions observed on the mountain (see Fig. 4) which were generally between westerly and southerly during this period. We must bear in mind, however, that this estimate of the drag is rather crude. The assumption that the horizontal pressure gradient is independent of position, for example, will almost certainly be invalid and this may introduce some errors. It is also possible that local features of the mountain will have affected the pressure measurements made at each station. Basing the drag estimate on measurements made at only three sites means that we cannot take this into account and hence the drag estimate might be biassed by the choice of the positions of the stations. More detailed experiments are needed to investigate this and to improve on the estimates of the drag.

Another source of error in this calculation might be that we have not correctly accounted for the differences in the heights of the stations when subtracting the average values of the pressure at low wind speeds. This error is most evident in the pressure differences between sites 1 and 2, where there is an almost constant offset between $p'_1 - p'_2$ and $\frac{1}{2}\rho (u_2^2 - u_1^2)$ (see Fig. 7). As noted in section 7 we do not have enough data during periods of low wind speed to improve upon our attempt to remove the mean pressure differences. However, for the purposes of estimating the drag, it seems reasonable to remove this error by making a correction to the pressure differences. This can be done by adding a constant offset to the pressure differences so that the mean value of $\Delta p'$ is equal to the mean value of the $\frac{1}{2}\rho u^2$ difference over the period of the experiment. This offset was applied to the $p'_1 - p'_2$ data only (the correction was $-0.36$ hPa) and the drag was then re-calculated using Eq. (14) as before. Figure 12 shows the resulting new estimate of the drag. In this case, the average drag exerted on the mountain is 3.6 Pa during the first 85 hours of the experiment. This is just over a half of the value obtained by using the uncorrected pressure data. From Fig. 12 we can see that this correction has largely removed the easterly bias in the drag vectors that was apparent in the initial calculation. The drag vectors now tend to a more northerly direction for the majority of the time. This direction is much closer to the wind directions measured on the mountain (see Figs. 4).
Figure 12. The drag vectors calculated using the first 85 hours of data which were available simultaneously from stations 1, 2 and 4 (see Table 1). The time shown on the horizontal axis refers to the time in hours after 0000 GMT on 20 November 1991. In the calculation of $\partial p/\partial x$ and $\partial p/\partial y$ a constant offset has been added to account for a possible incorrect removal of the component of the pressure differences due to the height differences between the instruments.

9. CONCLUSIONS

It seems that the measurement of the pressure differences across a mountain such as Black Combe is possible even though the mountain is relatively small (compared with more significant orography such as the Alps). Despite the limited spatial resolution given by only three reliable instruments and the short time period of the experiment, the available data reveal an interesting result, namely, the pressure differences appear to be between 1 and 2 hPa, and these differences correlate very well with the differences in $\frac{1}{2} \rho u^2$. During this experiment the incident flow was largely from the south-west. Ideally, we need to run an experiment such as this for a longer period of time so that the effect of more varied conditions can be studied. Problems with the instrumentation (such as data-logger failure) could also be avoided in the future by making the equipment more robust and protecting it further from severe wind and rain.

An average estimate of the drag, based on the pressure measurements made at stations 1, 2 and 4, is 6.4 Pa over the first 85 hours of the experiment. The calculated drag vectors, however, do not align closely with the wind direction measured at any of the stations and this appears to be due to an incorrect removal of the mean pressure differences between the pairs of instruments. Calculating the drag after making a correction to the pressure differences in an attempt to remove this effect gives an average value of 3.6 Pa with the force largely directed from the south. This is closer to the wind direction; it therefore seems likely that this estimate is more accurate. Without detailed study of the flow above the mountain surface it is impossible to say what mechanism is dominant in determining the drag. The high correlation of the pressure and $\frac{1}{2} \rho u^2$ differences is, however, consistent with gravity-wave generation.
ACKNOWLEDGEMENTS

We are indebted to Dr A. Ibbetson and Mr M. Cantwell of the University of Reading for their support and technical expertise in helping to set up and run the field experiment. We are grateful to Professor A. J. Thorpe (also of the University of Reading) for his encouragement and for valuable technical discussions. The experiment would not have been possible without the kind permission given by the land owner, Mr W. D. Park. We are most grateful to Mr Park for the interest he has shown in our work. Thanks are also due to the referees, whose suggestions have significantly improved the paper. S. B. Vosper was funded by a Science and Engineering Research Council CASE studentship in collaboration with the UK Meteorological Office.

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